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Control System Design Formulas

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List of 31 Control System Design Formulas

Control System Design

1) Angle of Asymptotes

$$\text{fx } \phi_k = \frac{(2 \cdot (\text{modulus}(N - M) - 1) + 1) \cdot \pi}{\text{modulus}(N - M)}$$

[Open Calculator !\[\]\(a870788d6ed9b8fd294b7654a8c8526b_img.jpg\)](#)

$$\text{ex } 5.834386\text{rad} = \frac{(2 \cdot (\text{modulus}(13 - 6) - 1) + 1) \cdot \pi}{\text{modulus}(13 - 6)}$$

2) Bandwidth Frequency given Damping Ratio

$$\text{fx } f_b = \omega_n \cdot \left(\sqrt{1 - (2 \cdot \zeta^2)} + \sqrt{\zeta^4 - (4 \cdot \zeta^2) + 2} \right)$$

[Open Calculator !\[\]\(c50c8b7b2cc2cf9ff925edec0ee94c0d_img.jpg\)](#)

$$\text{ex } 54.96966\text{Hz} = 23\text{Hz} \cdot \left(\sqrt{1 - (2 \cdot (0.1)^2)} + \sqrt{(0.1)^4 - (4 \cdot (0.1)^2) + 2} \right)$$

3) Damped Natural Frequency

$$\text{fx } \omega_d = \omega_n \cdot \sqrt{1 - \zeta^2}$$

[Open Calculator !\[\]\(f60b7a900783ac3fd531bfd9c111be6d_img.jpg\)](#)

$$\text{ex } 22.88471\text{Hz} = 23\text{Hz} \cdot \sqrt{1 - (0.1)^2}$$



4) Damping Ratio given Critical Damping 

$$\text{fx } \zeta = \frac{C}{C_c}$$

[Open Calculator !\[\]\(cbe80b694ebd74fcfe136a095b608235_img.jpg\)](#)

$$\text{ex } 0.100334 = \frac{0.6}{5.98}$$

5) Damping Ratio given Percentage Overshoot 

$$\text{fx } \zeta = - \frac{\ln\left(\frac{\%o}{100}\right)}{\sqrt{\pi^2 + \ln\left(\frac{\%o}{100}\right)^2}}$$

[Open Calculator !\[\]\(3e2231b1ad3ca8da8658228c00dd08e0_img.jpg\)](#)

$$\text{ex } 0.100106 = - \frac{\ln\left(\frac{72.9}{100}\right)}{\sqrt{\pi^2 + \ln\left(\frac{72.9}{100}\right)^2}}$$

6) Damping Ratio or Damping Factor 

$$\text{fx } \zeta = \frac{c}{2 \cdot \sqrt{m \cdot K_{\text{spring}}}}$$

[Open Calculator !\[\]\(0d5ec72f61334709c3fc9450209b754f_img.jpg\)](#)

$$\text{ex } 0.188147 = \frac{16}{2 \cdot \sqrt{35.45\text{kg} \cdot 51\text{N/m}}}$$



7) Delay Time 

$$fx \quad t_d = \frac{1 + (0.7 \cdot \zeta)}{\omega_n}$$

[Open Calculator !\[\]\(e78f798d4ea5c530c9db49e7d26e6b95_img.jpg\)](#)

$$ex \quad 0.046522s = \frac{1 + (0.7 \cdot 0.1)}{23Hz}$$

8) First Peak Overshoot 

$$fx \quad M_o = e^{-\frac{\pi \cdot \zeta}{\sqrt{1-\zeta^2}}}$$

[Open Calculator !\[\]\(05be7c7a8995decd503647c99211f7c2_img.jpg\)](#)


$$ex \quad 0.729248 = e^{-\frac{\pi \cdot 0.1}{\sqrt{1-(0.1)^2}}}$$

9) First Peak Undershoot 

$$fx \quad M_u = e^{-\frac{2 \cdot \zeta \cdot \pi}{\sqrt{1-\zeta^2}}}$$

[Open Calculator !\[\]\(fe3aebe81acea8d45108cd2768939da7_img.jpg\)](#)

$$ex \quad 0.531802 = e^{-\frac{2 \cdot 0.1 \cdot \pi}{\sqrt{1-(0.1)^2}}}$$

10) Gain-Bandwidth Product 

$$fx \quad G.B = \text{modulus}(A_M) \cdot BW$$

[Open Calculator !\[\]\(899d8b7697d64725bf017d3296cfcf1b_img.jpg\)](#)

$$ex \quad 56.16Hz = \text{modulus}(0.78) \cdot 72b/s$$



11) Number of Asymptotes 

$$fx \quad N_a = N - M$$

[Open Calculator !\[\]\(e2376d476d06eb31946dc01a69a4403a_img.jpg\)](#)

$$ex \quad 7 = 13 - 6$$

12) Number of Oscillations 

$$fx \quad n = \frac{t_s \cdot \omega_d}{2 \cdot \pi}$$

[Open Calculator !\[\]\(0b5e7e25e8775f7e7e80906ada4f0021_img.jpg\)](#)


$$ex \quad 6.365281\text{Hz} = \frac{1.748\text{s} \cdot 22.88\text{Hz}}{2 \cdot \pi}$$

13) Peak Time 

$$fx \quad t_p = \frac{\pi}{\omega_d}$$

[Open Calculator !\[\]\(bd3b31712ad9bab5a241210fa6925cdd_img.jpg\)](#)

$$ex \quad 0.137307\text{s} = \frac{\pi}{22.88\text{Hz}}$$

14) Peak Time given Damping Ratio 

$$fx \quad t_p = \frac{\pi}{\omega_n \cdot \sqrt{1 - \zeta^2}}$$

[Open Calculator !\[\]\(7bc43b319a082987e20f7bf78f4bab80_img.jpg\)](#)


$$ex \quad 0.137279\text{s} = \frac{\pi}{23\text{Hz} \cdot \sqrt{1 - (0.1)^2}}$$



15) Percentage Overshoot Open Calculator 

$$\text{fx } \%_o = 100 \cdot \left(e^{\frac{-\zeta \cdot \pi}{\sqrt{1 - (\zeta^2)}}} \right)$$

$$\text{ex } 72.92476 = 100 \cdot \left(e^{\frac{-0.1 \cdot \pi}{\sqrt{1 - (0.1)^2}}} \right)$$

16) Q-Factor Open Calculator 

$$\text{fx } Q = \frac{1}{2 \cdot \zeta}$$


$$\text{ex } 5 = \frac{1}{2 \cdot 0.1}$$

17) Resonant Frequency Open Calculator 

$$\text{fx } \omega_r = \omega_n \cdot \sqrt{1 - 2 \cdot \zeta^2}$$

$$\text{ex } 22.76884\text{Hz} = 23\text{Hz} \cdot \sqrt{1 - 2 \cdot (0.1)^2}$$



18) Resonant Peak 

$$\text{fx } M_r = \frac{1}{2 \cdot \zeta \cdot \sqrt{1 - \zeta^2}}$$

Open Calculator 

$$\text{ex } 5.025189 = \frac{1}{2 \cdot 0.1 \cdot \sqrt{1 - (0.1)^2}}$$

19) Rise Time given Damped Natural Frequency 

$$\text{fx } t_r = \frac{\pi - \Phi}{\omega_d}$$

Open Calculator 

$$\text{ex } 0.125507\text{s} = \frac{\pi - 0.27\text{rad}}{22.88\text{Hz}}$$

20) Rise Time given Damping Ratio 

$$\text{fx } t_r = \frac{\pi - \left(\Phi \cdot \frac{\pi}{180}\right)}{\omega_n \cdot \sqrt{1 - \zeta^2}}$$

Open Calculator 

$$\text{ex } 0.137073\text{s} = \frac{\pi - \left(0.27\text{rad} \cdot \frac{\pi}{180}\right)}{23\text{Hz} \cdot \sqrt{1 - (0.1)^2}}$$

21) Rise Time given Delay Time 

$$\text{fx } t_r = 1.5 \cdot t_d$$

Open Calculator 

$$\text{ex } 0.06\text{s} = 1.5 \cdot 0.04\text{s}$$



22) Setting Time when Tolerance is 2 Percent 

$$fx \quad t_s = \frac{4}{\zeta \cdot \omega_d}$$

Open Calculator 

$$ex \quad 1.748252s = \frac{4}{0.1 \cdot 22.88Hz}$$

23) Setting Time when Tolerance is 5 Percent 

$$fx \quad t_s = \frac{3}{\zeta \cdot \omega_d}$$

Open Calculator 


$$ex \quad 1.311189s = \frac{3}{0.1 \cdot 22.88Hz}$$

24) Steady State Error for Type 1 System 

$$fx \quad e_{ss} = \frac{A}{K_v}$$

Open Calculator 

$$ex \quad 0.064516 = \frac{2}{31}$$

25) Steady State Error for Type 2 System 

$$fx \quad e_{ss} = \frac{A}{K_a}$$

Open Calculator 

$$ex \quad 0.060606 = \frac{2}{33}$$



26) Steady State Error for Type Zero System

[Open Calculator !\[\]\(666e09182d4cd268646ea700ea60dcdf_img.jpg\)](#)

$$\text{fx } e_{ss} = \frac{A}{1 + K_p}$$

$$\text{ex } 0.060606 = \frac{2}{1 + 32}$$

27) Time of Peak Overshoot in Second Order System

[Open Calculator !\[\]\(003082e50e3009141f59bd5df831749f_img.jpg\)](#)

$$\text{fx } T_{po} = \frac{(2 \cdot k - 1) \cdot \pi}{\omega_d}$$

$$\text{ex } 1.235766\text{s} = \frac{(2 \cdot 5 - 1) \cdot \pi}{22.88\text{Hz}}$$

28) Time Period of Oscillations

[Open Calculator !\[\]\(d3102649f02e825ddb76dc3de0190154_img.jpg\)](#)

$$\text{fx } T = \frac{2 \cdot \pi}{\omega_d}$$

$$\text{ex } 0.274615\text{s} = \frac{2 \cdot \pi}{22.88\text{Hz}}$$



29) Time Response in Overdamped Case

fx

Open Calculator 

$$C_t = 1 - \left(\frac{e^{-\left(\zeta_{\text{over}} - \sqrt{\left(\zeta_{\text{over}}^2 - 1\right)}\right) \cdot \left(\omega_n \cdot T\right)}}{2 \cdot \sqrt{\left(\zeta_{\text{over}}^2 - 1\right)} - 1 \cdot \left(\zeta_{\text{over}} - \sqrt{\left(\zeta_{\text{over}}^2 - 1\right)}\right)} \right)$$

ex

$$0.807466 = 1 - \left(\frac{e^{-\left(1.12 - \sqrt{\left((1.12)^2 - 1\right)}\right) \cdot (23\text{Hz} \cdot 0.15\text{s})}}{2 \cdot \sqrt{\left((1.12)^2 - 1\right)} - 1 \cdot \left(1.12 - \sqrt{\left((1.12)^2 - 1\right)}\right)} \right)$$

30) Time Response in Undamped Case

fx

$$C_t = 1 - \cos(\omega_n \cdot T)$$

Open Calculator 

ex

$$1.952818 = 1 - \cos(23\text{Hz} \cdot 0.15\text{s})$$

31) Time Response of Critically Damped System

fx

$$C_t = 1 - e^{-\omega_n \cdot T} - \left(e^{-\omega_n \cdot T} \cdot \omega_n \cdot T \right)$$

Open Calculator 

ex

$$0.858732 = 1 - e^{-23\text{Hz} \cdot 0.15\text{s}} - \left(e^{-23\text{Hz} \cdot 0.15\text{s}} \cdot 23\text{Hz} \cdot 0.15\text{s} \right)$$



Variables Used






- $\%O$ Percentage Overshoot
- **A** Coefficient Value
- A_M Amplifier Gain in Mid Band
- **BW** Amplifier Bandwidth (*Bit Per Second*)
- **c** Damping Coefficient
- **C** Actual Damping
- C_c Critical Damping
- C_t Time Response for Second Order System
- e_{ss} Steady State Error
- f_b Bandwidth Frequency (*Hertz*)
- **G.B** Gain-Bandwidth Product (*Hertz*)
- **k** Kth Value
- K_a Acceleration Error Constant
- K_p Position of Error Constant
- K_{spring} Spring Constant (*Newton per Meter*)
- K_v Velocity Error Constant
- **m** Mass (*Kilogram*)
- **M** Number of Zeroes
- M_o Peak Overshoot
- M_r Resonant Peak
- M_u Peak Undershoot
- **n** Number of Oscillations (*Hertz*)




- **N** Number of Poles
- **N_a** Number of Asymptotes
- **Q** Q Factor
- **T** Time Period for Oscillations (Second)
- **t_d** Delay Time (Second)
- **t_p** Peak Time (Second)
- **T_{po}** Time of Peak Overshoot (Second)
- **t_r** Rise Time (Second)
- **t_s** Setting Time (Second)
- **ζ** Damping Ratio
- **ζ_{over}** Overdamping Ratio
- **Φ** Phase Shift (Radian)
- **Φ_k** Angle of Asymptotes (Radian)
- **ω_d** Damped Natural Frequency (Hertz)
- **ω_n** Natural Frequency of Oscillation (Hertz)
- **ω_r** Resonant Frequency (Hertz)



Constants, Functions, Measurements used

- **Constant:** **pi**, 3.14159265358979323846264338327950288
Archimedes' constant
- **Constant:** **e**, 2.71828182845904523536028747135266249
Napier's constant
- **Function:** **cos**, $\cos(\text{Angle})$
Cosine of an angle is the ratio of the side adjacent to the angle to the hypotenuse of the triangle.
- **Function:** **ln**, $\ln(\text{Number})$
The natural logarithm, also known as the logarithm to the base e, is the inverse function of the natural exponential function.
- **Function:** **modulus**, modulus
Modulus of a number is the remainder when that number is divided by another number.
- **Function:** **sqrt**, $\sqrt{\text{Number}}$
A square root function is a function that takes a non-negative number as an input and returns the square root of the given input number.
- **Measurement:** **Weight** in Kilogram (kg)
Weight Unit Conversion 
- **Measurement:** **Time** in Second (s)
Time Unit Conversion 
- **Measurement:** **Angle** in Radian (rad)
Angle Unit Conversion 
- **Measurement:** **Frequency** in Hertz (Hz)
Frequency Unit Conversion 
- **Measurement:** **Bandwidth** in Bit Per Second (b/s)
Bandwidth Unit Conversion 



- **Measurement: Stiffness Constant** in Newton per Meter (N/m)
Stiffness Constant Unit Conversion 



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